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Minimum Weight Design of Rotorcraft Blades with Multiple Frequency and Stress Constraints

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Introduction

AN important consideration in helicopter rotor blade design is to reduce vibration without increasing blade weight. Rotor blade vibration can be reduced by separating its natural frequencies from the harmonics of the airloads or the excitation frequencies to avoid resonance. In the conventional design process this is usually done by post-design addition of nonstructural masses, which often leads to weight penalties. Today, one of the more promising design approaches is the application of optimization techniques during the design process to reduce vibration.[1,5] Some recent work is devoted to reducing vibration of modal shaping[1] or by controlling the vertical hub shears and moments.[2] An early attempt at optimum blade design for proper placement of natural frequencies is due to Peters,[3] who addresses the optimum design of a rectangular blade with frequency and autorotational inertia constraint. Frequency placement alone was also addressed in Ref. 4 using the optimality criteria approach. In the problem addressed in this Note, blade weight is the objective function and constraints are imposed on natural frequencies, autorotational inertia, and centrifugal stress. This is an extension of the work of Ref. 5, where only the frequencies of first flapping dominated and first lead-lag dominanted blade modes were constrained, without any constraint on blade stress.

Blade Model

The blade model includes a nonuniform box beam located inside the airfoil. Its total weight W has two components, W_b and W_o, where W_b denotes the box beam weight and W_o represents the nonstructural weight of the blade, including the weight of the skin, honeycomb, etc., along with the weight of the tuning masses added to the blade. The blade is discretized into finite segments and the blade weight in discretized form is given as

$$W = \sum_{J=1}^{N} \rho_j A_j L_j + \sum_{J=1}^{N} W_{o_j} \tag{1}$$

where N denotes the total number of segments and p_j, A_j, L_j, and W_{o_j} denote the density, the cross sectional area, the length, and the nonstructural weight of the jth segment, respectively. The autorotational inertia (AI) of the blade is calculated as

$$AI = \sum_{J=1}^{N} W_j r_j^2 \tag{2}$$

Presented as Paper 88-2337 at the AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, Williamsburg, VA, April 18–20, 1988; received May 21, 1988; revision received March 28, 1989.

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where W_j is the total weight and r_j is the distance from the root to the center of the jth segment. The expression for the blade stress is

$$\alpha_i = \sum_{j=1}^{N} M_j \Omega^2 r_j / A_i \tag{3}$$

where σ_i is the stress due to centrifugal forces, A_i the cross-sectional area of the ith segment, M_j the total mass of the jth segment, and Ω the blade rpm.

Optimization Formulation

The optimization problem is posed as follows:

$$\text{minimize } W(\phi)$$

where weight W is given by Eq. (1) and ϕ denotes the vector of design variables, subject to the normalized constraints

$$g_k(\phi) = (f_k/f_{k_U}) - 1 \le 0 \qquad k = 1,2,\ldots,5 \tag{4}$$

$$g_{k+5}(\phi) = 1 - (f_k/f_{k_L}) \le 0 \qquad k = 1,2,\ldots,5 \tag{5}$$

$$g_{11}(\phi)1 - (AI/\alpha) \le 0 \tag{6}$$

$$g_{11+k}(\phi) = 1 - \sigma_{\max}/(\sigma_k FS) \le 0 \qquad k = 1,2,\ldots,N \tag{7}$$

and side constraints

$$\phi_{i_L} \le \phi_i \le \phi_{i_U} \tag{8}$$

In Eqs. (4) and (5), the frequencies associated with the first five elastic modes of coupled vibration are denoted by f_1, f_2, f_3, f_4, and f_5 (includes three lead-lag and two flapping) and f_{k_U}, f_{k_L} denote the upper and lower bounds on the kth frequency f_k. In Eq. (6), α represents the minimum prescribed autorotational inertia value. In Eq. (7), σ_k is the stress in the kth segment given by Eq. (3), $\sigma_{\max}$ the maximum allowable stress in the blade and FS a factor of safety. In Eq. (8), ϕ_i denotes the ith design variable and ϕ_{i_U} and ϕ_{i_L} represent the associated upper and lower bounds, respectively. The design variable include box beam dimensions, taper ratio, and magnitudes of the nonstructural weights located inside the box beam.

The modal analysis portion of the program CAMRAD,[6] which uses a modified Galerkin approach, is used for calculating the frequencies. The general-purpose optimization program CONMIN[7] and a linear approximation technique that uses Taylor series expansion is used for optimization. A sensitivity analysis part of the procedure, and analytical expressions obtained for the derivatives of the objective function, the autorotational inertia constraint, and the stress constraints. A central-difference scheme is used for the derivative of the frequency constraints.

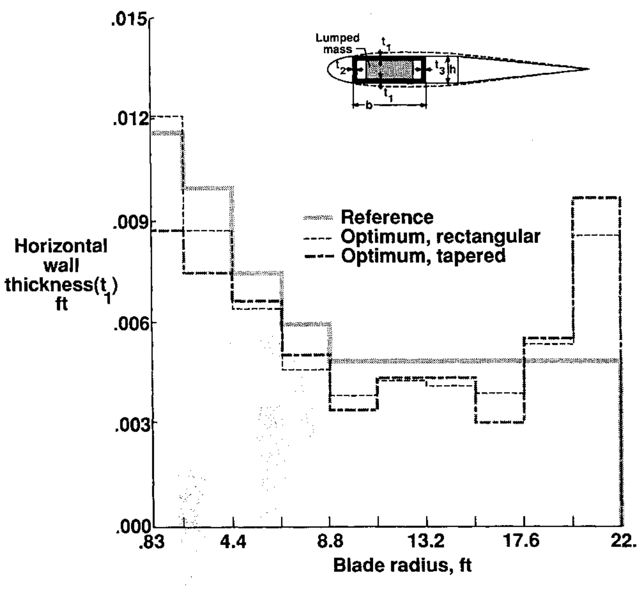

Fig. 1 Box beam horizontal wall thickness (t_1) distribution; reference, optimum.

Test Problem

The reference blade (Fig. 1) is articulated and has a rigid hub. It has a rectangular planform, a pretwist, and a root spring that allows torsional motion. A box beam with unequal vertical wall thickness is located inside the airfoil. It is assumed that the box beam contributes to the blade stiffness, and that the contributions of the skin, honeycomb, etc., to the blade stiffness are neglected. The details for calculating the box beam section properties can be found in Ref. 5. The properties of the box beam located inside the airfoil (Fig. 1) are: $h = 0.117$ ft, $b = 0.463$ ft, $\rho = 8.645$ slug/ft^3, $E = 2.340 \times 10^9$ lb/ft^2, $\sigma_{\max} = 1.93 \times 10^7$ lb/ft^2, and $FS = 3$. The blade is discretized into 10 segments and details of the blade segment data are presented in Refs. 3 and 8. The frequencies of interest of the reference blade are presented in Table 1. The first three lead-lag and the first two flapping dominated modes are away from the critical frequencies and need not be improved further. Therefore, the frequency windows for the optimum blade are set to be within $\pm 1\%$ of these values.

Results and Discussion

Some typical results obtained by applying the optimization procedure to the design of both rectangular and tapered rotor blades are presented in Tabler 1 and Figs. 1–3. Further results, can be found in Ref. 8. In the case of the rectangular blade, the taper ratio λ is held constant, i.e., $\lambda = 1.0$. In all cases, convergence is typically achieved in 8–10 cycles (full analysis). A convergence tolerance of 0.5×10^{-5} is used for three consecutive values of the objective function in this study. Table 1, column 1 represents the reference blade data. Columns 2 and 3 give the corresponding information for the optimum design with constraints on five frequencies, autorotational inertia, and centrifugal stress for the rectangular and tapered blades, respectively. The table indicates that the optimum rectangular blade is 2.67 and 4.74% lighter than the reference blade, and the optimum tapered blade is 6.21% lighter than the reference blade. The first lead-lag frequency (f_1) is at its prescribed upper bound after optimization and the autorotational inertia constraint is active in all cases. Some typical design variable distributions are presented in Figs. 1–3. Fig. 1 presents the optimum and the reference blade box beam horizontal wall thickness (t_1) distributions along the blade span for the rectangular and tapered blades. In both cases, the optimum blade has a larger value of t_1 than the reference blade at the blade tip and in case of the tapered blade the value of t_1 at the blade root for the optimum blade is much smaller than the reference blade value. The larger design variable values toward the blade tip are

Table 1 Optimization results

	Frequency bounds			Optimum	
	Upper	Lower	Reference	Rectangular	Tapered
f_1 (Hz)	12.162	12.408	12.285	12.408	12.408
f_2 (Hz)	15.936	16.258	16.098	16.075	16.066
f_3 (Hz)	20.704	21.122	20.913	21.081	20.888
f_4 (Hz)	34.272	34.966	34.62	34.823	34.678
f_5 (Hz)	35.502	36.219	35.861	35.507	35.507
λ_h			1.0	1.0	1.490
Autorotational inertia (lb-ft^2)			571.3	571.3	517.3
Blade weight (lb)			98.27	93.613	92.16
Percent reduction in blade weight[a]			–	4.74	6.212

[a]From reference blade.

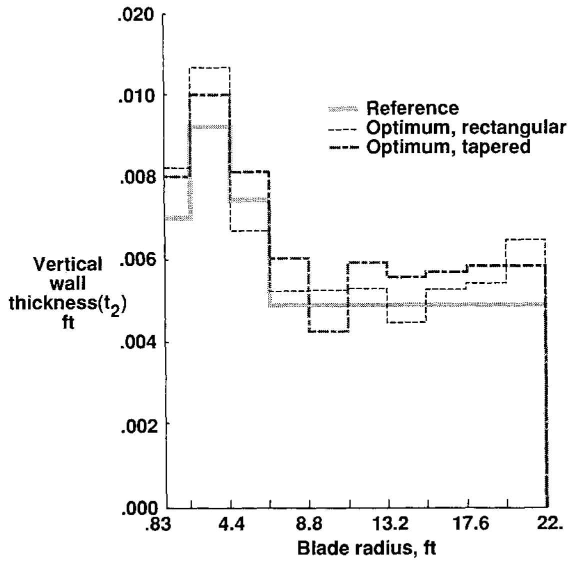

Fig. 2 Box beam vertical wall thickness (t_2) distributions; reference, optimum.

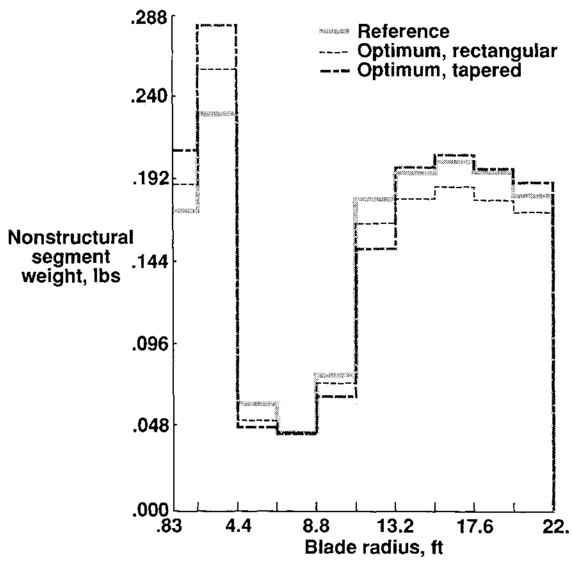

Fig. 3 Nonstructural segment weight distributions; reference, optimum.

caused by the presence of the autorotational inertia constraint which encourages the addition of mass at locations outboard. Figure 2 presents the box beam vertical wall thickness (t_2) distributions for the reference and the optimum blades. Once again, the autorotational inertia constraint plays a big role in increasing the value of t_2 at blade tip for the rectangular and the reference blades. However, the increase is not as significant as in the case of t_1. The nature of these thickness distributions are also different, as t_1 primarily affects the flapping frequency and t_2 the lead-lag frequency. Figure 3 depicts the optimum and the reference blade nonstructural segment weight distributions along the blade radius for the rectangular and tapered blades. For the rectangular blade the optimum blade has lower nonstructural weight throughout the blade span. However, for the tapered blade the optimum blade has larger nonstructural weight towards the tip than the reference blade. This is because the blade is being tapered and has reduced structural weight at the blade tip. Therefore, in order to satisfy the autorotational inertia constraint, the nonstructural weight at the tip must increase. Even so, the total weights of the optimum blade are still lower than the reference blade.

A comparison of the results of the study with those obtained in Ref. 5 shows that, with an increase in the number of constraints, the optimum blade weight increases along with significant changes in the design variable distributions for both the rectangular and the tapered blades.

Concluding Remarks

A procedure is described for the minimum weight design of helicopter rotor blades with constraints on multiple coupled flag-lag natural frequencies, autorotational inertia, and centrifugal stress. Optimum designs of blades with both rectangular and tapered planforms are obtained in typically 8–10 cycles.

The results of the study indicate that a significant reduction in blade weight is possible, even with constraints on multiple natural frequencies, for both the rectangular and the tapered blades. The optimization process tends to redistribute mass towards the blade tip due to the presence of the autorotational inertia constraint. The inclusion of multiple frequency constraints affects the optimum blade weight as well as the design variable distributions.

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